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Black hole mergers: do gas discs lead to spin alignment?

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ABSTRACT
In this Letter we revisit arguments suggesting that the Bardeen–Petterson effect can coalign the spins of a central supermassive black hole binary accreting from a circumbinary (or circumnuclear) gas disc. We improve on previous estimates by adding the dependence on system parameters, and noting that the nonlinear nature of warp propagation in a thin viscous disc affects alignment. This reduces the disc's ability to communicate the warp, and can severely reduce the effectiveness of disc–assisted spin alignment. We test our predictions with a Monte Carlo realization of random misalignments and accretion rates and we find that the outcome depends strongly on the spin magnitude. We estimate a generous upper limit to the probability of alignment by making assumptions which favour it throughout. Even with these assumptions, about 40% of black holes with $a \gtrsim 0.5$ do not have time to align with the disc. If the residual misalignment is not small and it is maintained down to the final coalescence phase this can give a powerful recoil velocity to the merged hole. Highly spinning black holes are thus more likely of being subject to strong recoils, the occurrence of which is currently debated.

Key words: accretion, accretion discs - black hole physics - galaxies: evolution - galaxies: nuclei

1 INTRODUCTION

The cosmic evolution of the spin of supermassive black holes (SMBH) residing in galaxy centers has several important astrophysical implications. By regulating the efficiency of conversion of matter into radiation, ϵ, the spin magnitude a effectively determines the ability of black holes to grow rapidly by accretion. Defining the Eddington time as

$$t_{\rm Edd} = \frac{\kappa c}{4\pi G} \approx 4.5\times10^{8}{\rm yrs}, \quad (1)$$

where κ is the Thomson opacity, c the speed of light and G the gravitational constant, a black hole accreting continuously at the Eddington rate grows in mass exponentially on a timescale $\tau = \epsilon t_{\rm Edd}/(1-\epsilon)$. Rapidly spinning black holes have $\epsilon \approx 0.5$, and thus $\tau \approx t_{\rm Edd}$, while for non-rotating black holes $\tau \approx 0.06 t_{\rm Edd}$. If most holes spin rapidly, the existence of luminous quasars at redshift $\gtrsim 7$, when the age of the Universe was $\sim 2t_{\rm Edd}$, (Fan et al. 2004; Fan et al. 2006; Mortlock et al. 2011) requires very massive ($\gtrsim 10^5 {\rm M}_\odot$) seed black holes (e.g. Volonteri & Rees 2005), that might form through direct collapse of primordial gas (Lodato & Natarajan 2006). Alternatively, King & Pringle (2006, 2007) and King et al. (2008) argued instead that the spins (and so ϵ) could remain relatively low, if most SMBH accrete through a sequence of randomly oriented accretion events, in what is known as the chaotic accretion picture.

Black hole spin is also important in a completely different context. The relative orientation of spins during binary black hole coalescence determines the waveform of the gravitational radiation that may be detected by planned gravitational wave observatories such as NGO-eLISA. Moreover, when the spins of the two merging black holes are significantly misaligned to each other, the remnant black hole can receive recoil velocities up to $\sim$ 4000 km/sec (Campanelli et al. 2007a,b; Lousto & Zlochower 2011). There have been claims of the detection of several fast recoiling black holes via their emission lines (Komossa et al. 2008; Civano et al. 2012), but the interpretation is unclear (Bogdanović et al. 2009; Dotti et al. 2009a; Blecha et al. 2012) (see Komossa 2012 for a review). The recoil can also lead to a significant brightening of the accretion disc (Schnittman & Krolik 2008; Rossi et al. 2010; Corrales et al. 2010; Anderson et al. 2010) to near–Eddington luminosities, either in the infrared (Schnittman & Krolik 2008) or at higher frequencies (UV to X-rays) if the radiation is not thermalized (Rossi et al. 2010).

Black hole mergers are expected to occur in gas rich environments, and circumbinary gas discs may play a key role in bringing the binary to separations close enough for gravitational wave emission to drive the final coalescence. The effectiveness of such gas–assisted mergers is debated. Galaxy–scale simulations appear to imply that in the presence of gas a black hole pair might reach separations of the order of 0.01 pc and form a binary within $1-5\times10^7$ years (Escala et al. 2005; Dotti et al. 2009b). However, higher–resolution computations (Lodato et al. 2009) show that, at distances of the order of 0.01 pc, the circumbinary disc probably becomes self–gravitating and fragments, severely limiting the possibility of

mergers within a Hubble time. In contrast, in the context of the chaotic accretion picture of King & Pringle (2006), Nixon et al. (2011a) have shown that a sequence of accretion episodes where the circumbinary disc can be either co– or counter–aligned with the binary is much more effective than a simple prograde disc in bringing the binary to coalescence.

Gas discs can also affect the mutual orientation of the spins of the two black holes through the Bardeen–Petterson (BP) effect (Bardeen & Petterson 1975; Papaloizou & Pringle 1983). Lense–Thirring precession and viscous torques in a disc around a black hole tend to co– or counteralign its spin with the disc axis (Lodato & Pringle 2006; King et al. 2005). In a binary black hole system, each hole may have an individual disc around it, whose initial orientation agrees with that of the circumbinary disc. Then the BP effect can co– or counteralign the two spins. Clearly, for this to work, the orientation of the circumbinary disc, which is feeding the individual black hole discs, must stay roughly constant. (This does not hold in the chaotic accretion picture, cf Nixon et al. 2011a,b; Nixon 2012.)

Bogdanović et al. (2007) (hereafter BRM) make order–of–magnitude estimates of the BP effect. They consider the alignment timescale for a single black hole and its accretion disc. BRM find that this is very short, and conclude that each black hole is effectively aligned with its disc and so that the two black hole spins are each co– or counter–aligned with the common circumnuclear disc at the time when they form a binary. Perego et al. (2009) improve their study finding slightly longer timescale. The numerical simulations of Dotti et al. (2010) appear to support this picture, although with some differences, depending on whether the circumnuclear disc is warm or cold, with the coldest (and thinnest) disc providing more effective alignment.

The estimates of BRM have a number of shortcomings. First, as mentioned above, their argument assumes that the discs surrounding each black hole share their orientations with the circumbinary disc, and that this does not change with time. If the accretion process during the merger is chaotic, both assumptions may be invalid. But even within the framework where the disc orientations track the binary orbit, the alignment timescale may not be as short as implied by BRM. Indeed, BRM evaluate the accretion timescale is by considering the flow properties at the Bondi radius, at a distance of 40 pc from the hole. However, the viscous time at these distances from the hole is much larger than a Hubble time for a thin disc, and still larger than the binary evolution timescale if the disc is assumed to be thick. Thus, such estimates may not accurately describe the flow properties at the warp radius, located at a few hundreds Schwarzschild radii. Secondly, and most importantly, both BRM and Perego et al. (2009) assume that the warp diffusion coefficient, usually called ν_2, is given by $\nu_2/\nu_1 = 1/2\alpha^2$, where ν_1 is the disc viscosity and α the standard Shakura-Sunyaev parameter (Shakura & Sunyaev 1973). This is correct only to first order for small amplitude warps (Papaloizou & Pringle 1983), while large amplitude warps diffuse much less efficiently (Ogilvie 1999; Lodato & Price 2010) (note also that the simulations by Dotti et al. 2010 assume that the warp diffusion coefficient is independent of amplitude). It is thus possible that large initial misalignments stay misaligned throughout the merger.

In this Letter we revisit the arguments of BRM and calculate a lower limit to the probability that the black hole spin remains misaligned (i.e. it does not have sufficient time to reach the aligned configuration) with its own disc at the point when gravitational wave reaction begins to control the merger process. In such a case it is very unlikely that the the two spins in a binary will end up aligned unless the initial misalignment is very small. We repeat that even in cases where we predict local alignment, a mutual coalignment of the two spins is only possible if the merger does not occur through a sequence of chaotic accretion events.

The paper is organized as follows: in section 2 we describe the basic assumptions of our model and the procedure we adopt to compute the alignment probability. In section 3 we discuss our main results for a suitable choice of parameters, and in section 4 we draw our conclusions.

2 MODELING SPIN ALIGNMENT

Natarajan & Pringle (1998) estimate the alignment timescale for the BP effect in the linear case where $\nu_2/\nu_1 = 1/2\alpha^2$. BRM use these results to estimate spin alignment. In general, one can define an α_2 coefficient corresponding to the diffusion coefficient ν_2, such that $\nu_2/\nu_1 = \alpha_2/\alpha$. The value of α_2 can be computed based on the nonlinear theory of warp propagation of Ogilvie (1999). This uses conservation laws to work out the connections between the viscosity coefficients, assuming that the local viscosity is isotropic. The validity of this theory has been demonstrated numerically by Lodato & Price (2010). For small warps, we have:

$$\frac{\alpha_2}{\alpha} = \frac{1}{2\alpha^2}\frac{4(1+7\alpha^2)}{4+\alpha^2}. \tag{2}$$

The above equation is, however, inadequate for large amplitude warps. Ogilvie (1999) provides a slowly converging Taylor series including additional amplitude dependent terms, but the full nonlinear result must be computed numerically.

The warp amplitude is measured by the parameter $\psi = \mathrm{d}\beta/\mathrm{d}\ln R$, where β is the local inclination of the disc. To compute the warp evolution accurately, one would thus not only need to know the misalignment between the outer disc axis and the black hole spin, θ, but also how steep its gradient is. This clearly requires a detailed calculation of the shape of the disc. For our preliminary assessment of the effect, we simply assume that the warp is a smooth function, varying over a distance of the order of the disc size, and so approximately $\psi \approx \theta$. This is a conservative assumption, since a steeper warp profile would increase the warp amplitude and thus reduce further the effectiveness of warp propagation. Indeed for sufficiently high inclination or low viscosity the disc may break, i.e. make a sharp transition between two distinct planes (Nixon & King 2012a).

The alignment timescale for small warps and a steady state disc is given by (e.g. Scheuer & Feiler 1996; Natarajan & Pringle 1998; Lodato & Pringle 2006) as

$$t_{\rm align} = 3a\frac{\nu_1}{\nu_2}\frac{M}{\dot{M}}\left(\frac{R_{\rm S}}{R_{\rm w}}\right)^{1/2}, \tag{3}$$

where M is the black hole mass, a the hole spin parameter, $\dot{M}$ the accretion rate through the disc, $R_{\rm S} = 2GM/c^2$ is the Schwarzschild radius, and $R_{\rm w}$ is the warp radius. In (3) the warp radius $R_{\rm w}$ is taken as the point at which the warp diffusion timescale equals the Lense–Thirring timescale, so that

$$R_{\rm w} = \frac{2GJ_{\rm h}}{c^2\nu_2}, \tag{4}$$

where $J_{\rm h} = aGM^2/c$ is the black hole angular momentum. The ratio $R_{\rm w}/R_{\rm S}$ is given by (cf. the analogous expression in Natarajan & Pringle 1998):

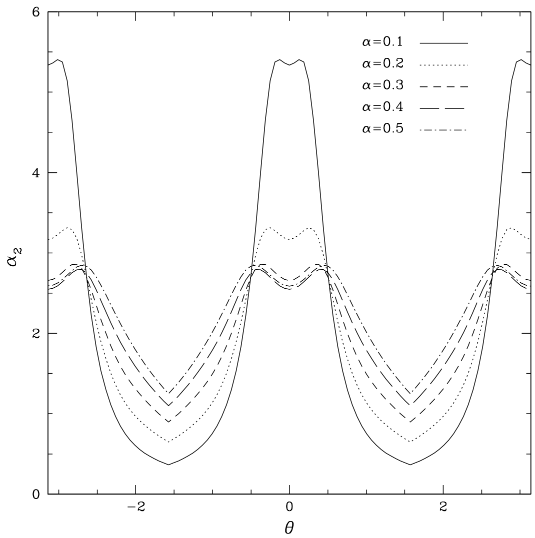

Figure 1. The value of the diffusion coefficient α_2 as a function of the misalignment θ for several choices of α=0.1 (solid line), 0.2 (dotted line), 0.3 (dashed line), 0.4 (long-dashed line) and 0.5 (dot-dashed line). While for small misalignments ($\theta \approx 0$), α_2 can be quite large, its value drops significantly (even below unity) when the misalignment approaches $\pi/2$.

$$\frac{R_{\rm w}}{R_{\rm S}} = \frac{1}{2^{1/3}}\left(\frac{a}{\alpha_2}\right)^{2/3}\left(\frac{H}{R}\right)^{-4/3}, \tag{5}$$

where H/R is the aspect ratio of the disc. Inserting this in equation 3 and scaling the accretion rate in units of the Eddington value $\dot{M}_{\rm Edd} = \epsilon M/t_{\rm Edd}$ we obtain:

$$t_{\rm align} \approx 3.37\alpha\left(\frac{a}{\alpha_2}\right)^{2/3}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{-1}\left(\frac{H}{R}\right)^{2/3}\epsilon t_{\rm Edd} \approx \tag{6}$$
$$7\times10^6\left(\frac{a}{\alpha_2}\right)^{2/3}\left(\frac{\alpha}{0.1}\right)\left(\frac{\dot{M}}{0.1\dot{M}_{\rm Edd}}\right)^{-1}\left(\frac{H/R}{0.01}\right)^{2/3}\left(\frac{\epsilon}{0.1}\right)\,{\rm yr},$$

where the various parameters have been scaled to typical values. Note that this timescale is only a factor 2 smaller than the typical estimates for the shrinking time in a gas-rich environment, which is $\sim 10^7$ years (Escala et al. 2005; Dotti et al. 2009b). For small warps, α_2 can become very large, so the alignment timescale is much smaller than the shrinking time. The same also occurs for slowly spinning black holes, for which both a and the accretion efficiency ϵ are small. Conversely, for maximally spinning black holes, $a \approx 1$, for which $\epsilon \approx 0.4$, alignment for large warps might require a timescale longer than the shrinking time.

Natarajan & Pringle (1998) also evaluate H/R as a function of the main disc parameters (black hole mass and accretion rate). However, the dependence turns out to be rather weak, and so we have just chosen to keep that number fixed at a representative value of 0.01. In (6) we scaled the Eddington ratio $\dot{M} = 0.1\dot{M}_{\rm Edd}$, which is typical for accreting black holes in AGN. Also this assumption is favourable for the alignment process since it is expected that BH in binary systems accrete ad a reduced rate due to binary torques (typically at 10 per cent of the unperturbed rate, Artymowicz & Lubow 1996) and because fragmentation in the outer disc might lead to significant star formation (Lodato et al. 2009). However, in our calculation we explore a wider range of $\dot{M}$.

In our calculation we take into account the relationship between the spin parameter a and the accretion efficiency $\epsilon(a)$, based on the formalism of Bardeen (1973) (cf. King et al. 2008, their fig. 5):

$$\epsilon = 1 - \frac{r_{\rm isco}^{3/2} - 2r_{\rm isco}^{1/2} \pm a}{r_{\rm isco}^{3/4}\left(r_{\rm isco}^{3/2} - 3r_{\rm isco}^{1/2} \pm 2a\right)^{1/2}}, \tag{7}$$

where $r_{\rm isco} = R_{\rm isco}c^2/GM$ is the radius of the innermost stable orbit in units of the gravitational radius and is given by

$$R_{\rm isco} = \frac{GM}{c^2}\left[3 + Z_2 \mp (3 - Z_1)^{1/2}(3 + Z_1 + 2Z_2)^{1/2}\right], \tag{8}$$

$$Z_1 = 1 + \left(1 - a^2\right)^{1/3}\left[(1 + a)^{1/3} + (1 - a)^{1/3}\right], \tag{9}$$

$$Z_2 = \left(3a^2 + Z_1^2\right)^{1/2}, \tag{10}$$

and the upper/lower sign refers to particle orbiting in prograde/retrograde orbits with respect to the rotation of the hole.

Finally, for any given misalignment θ, we compute $\alpha_2(\alpha,\theta)$ numerically, using the full nonlinear theory of Ogilvie (1999). We plot in Fig. 1 the resulting value of α_2 as a function of θ for various choices of α, which shows that while for small misalignments ($\theta \approx 0$), α_2 can be quite large, its value drops significantly (even below unity) when the misalignment approaches $\pi/2$.

3 RESULTS

We compute the probability of alignment through a Monte Carlo realization of $N = 10^4$ events. The initial angles θ are randomly distributed between $-\pi$ and π, thus including also cases where the disc and the black hole end up in a counteraligned configuration. We generate the angles uniformly in $\cos\theta$ to have an isotropic distribution in three dimensions. For each event we generate a random value of the accretion rate $\dot{M}/\dot{M}_{\rm Edd}$ between 10^{-4} and 1 (the same range as explored by Perego et al. 2009), with uniform logarithmic distribution. For each choice of the remaining two free parameters α and a, we thus obtain the probability distribution of $t_{\rm align}$: we compute this distribution varying over 5 different values of α and 100 different values of a. Note that, for large θ and small α, the theory of Ogilvie (1999) would fomally predict a negative azimuthal viscosity coefficient. The behaviour of the disc in such cases is unclear, but since the viscosity must already have passed through small positive values it is likely that it has in reality broken into two distinct planes, thus making alignment even slower (Lodato & Price 2010; Nixon & King 2012a). When this occurs, we simply assume that the real alignment time becomes much larger than any other timescale in the problem.

We plot in Fig. 2 with black lines the probability that the alignment time is smaller than 10 Myrs (left panel) and 50 Myrs (right panel) as a function of the spin parameter a, for various choices of α (with the same notation as Fig. 1). The values of 10 and 50 Myrs are taken as proxies for the shrinking timescale of the binary (Escala et al. 2005; Dotti et al. 2009b). The plots demonstrate the general features already discussed in section 2. For small spin parameters alignment is indeed efficient and we expect most spins to align or counteralign with their discs by the time the binary approaches coalescence. However this is not true for larger spins. In particular, for $a > 0.5$ we expect a sizeable fraction of the systems, of the order of 30-40% (or even more for extreme Kerr black holes), to not reach the aligned configuration after 10 Myrs. The result is still present, for 50 Myrs, in which case obviously the black holes have more time to become aligned.

The probability distributions computed with $\alpha = 0.1$ are quite

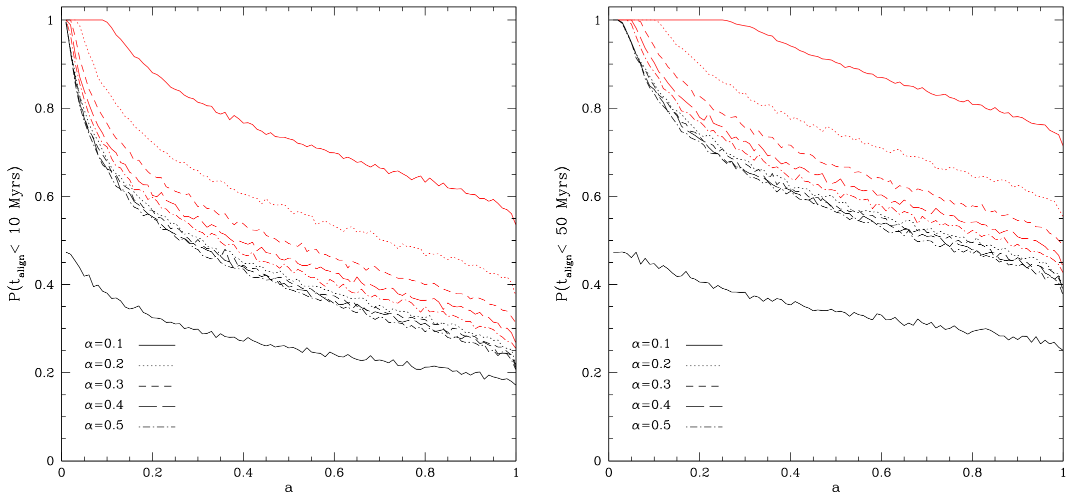

Figure 2. Probability that the alignment time $t_{\rm align}$ is smaller than 10 Myrs (left panel) or 50 Myrs (right panel), as a function of the spin parameter a, for various choices of α, using the same notation as in Fig. 1. For large values of a a sizeable fraction of systems are likely to have residual spin misalignments after a binary shrinking time. The red lines show the corresponding probabilities in the case where we use Eq. (2) to compute α_2 and we thus do not consider the effect of the non-linearity of the warp on the diffusion coefficients. In this case, there is a much stronger dependence on α and for small α alignment is much more efficient.

different from the others, because in this case we completely remove from the alignment process all the events with negative viscosity coefficient. As noted above, Ogilvie (1999) predicts this behavior for small values of α. We note however that even such a strong assumption does not affect significantly the distributions for large value of a where the effects described above are definitely more important.

For all the other values of α, there is no negative viscosity event: we note that our results are almost independent on α and that they are not very sensitive to the proxy chosen (10 or 50 Myrs), ensuring that our results are robust with respect to different choices of the merger timescale.

In Fig. 2 we also plot with red lines (and using the same line styles as above) the corresponding probabilities in the case where we use the warp diffusion coefficient in the linear regime (Eq. 2) (Ogilvie 1999; Lodato & Price 2010) and we thus do not consider the effect of the non-linearity of the warp on the diffusion coefficients. The effect of the non-linearity is clearly very significant. In particular, it turns out that in the linear case the alignment probability has a much stronger dependence on α (essentially because in the non-linear case the range of values of α_2 is much smaller than in the linear case) and alignment is indeed much faster, especially for low α.

We have also considered simpler cases where we keep the Eddington ratio fixed, in order to evaluate more clearly the dependance on this parameter. Clearly, since the alignment time is inversely proportional to $\dot{M}$, larger Eddington ratios (such as those assumed by BRM) imply a faster alignment. However, also in these cases, we find that alignment within 10Myrs occurs only for $a < 0.6$ and $a < 0.3$ for the two cases where $\dot{M} = 0.1\dot{M}_{\rm Edd}$ and $\dot{M} = 0.01\dot{M}_{\rm Edd}$, respectively. In order to have fast alignment for high spins the accretion rate is required to stay at the Eddington level for a prolonged period of time.

4 DISCUSSION AND CONCLUSIONS

We have revisited the arguments suggesting that gas disc are effective in bringing the spins of the two black holes in a merging binary into alignment. In particular, we have improved on previous estimates (BRM) by taking into full consideration how the alignment timescale changes with the system parameters. In addition, we have also included the reduction in the warp diffusion coefficient when the misalignment angle becomes large (Ogilvie 1999), an effect previously neglected.

Contrary to previous claims, if the black holes are rapidly spinning ($a \gtrsim 0.5$), the system is would not end up to be completely aligned, in up to 40% of the cases, at the time at which the two holes are brought together at distances of the order of 0.01 pc (the current resolution limit for numerical simulations of the process).

Our results are consistent, at least within orders of magnitude, with the previous investigation by Perego et al. (2009). In particular, we find a similar dependence of the alignment timescale with respect to the spin of the black hole and the Eddington ratio. Contrary to Perego et al. (2009), our estimates show that the alignment timescale is a strong function of the initial misalignment angle. This comes from having considered the full non linear warp propagation theory, instead of restricting to the small amplitude regime: this is a key element, because it allow us to compute a probability for the alignment process, based on the expected distribution of initial misalignments.

Highly spinning black holes are thus more likely to maintain a significant misalignment, which causes a high recoil velocity. The current lack of observational evidence for strongly recoiling black holes (but see the recent case of CID-42, Civano et al. 2012) suggests that the average black hole spin is rather low, in line with the predictions of the chaotic accretion picture (King & Pringle 2006). We note that for $a \lesssim 0.6$, as suggested above, the accretion efficiency is $\epsilon \simeq 0.1$. This is in line with the values suggested by the Soltan argument (Soltan 1982; Yu & Tremaine 2002) relating the average SMBH mass to the radiation background of the universe

(see also detailed discussion in King et al. 2008). We stress here that our conclusion regarding the magnitude of the black hole spin is related to the average properties of the BH population. Individual SMBH might well have large spins, and observations of broad iron lines (e.g., Brenneman & Reynolds 2006) would naturally be biased in their favour. On the contrary, if recoiling black holes are found to be more common, such limitation on the magnitude of the black hole spin would not apply.

Our model is certainly very idealized and could be refined in several ways. First of all, we have used a uniform logarithmic distribution of the Eddington value without considering in a detailed way how do tidal effects and gap opening (Artymowicz & Lubow 1996) affect the accretion rate on the binary elements. Another interesting effect arises from the fact that the alignment timescale scales inversely with the accretion rate. In binaries with a large mass ratio, where accretion occurs preferentially onto the secondary, the primary might be much harder to align, with potentially interesting effects on gravitational wave emission.

A second important limitation of our work comes from the assumption that the lengthscale over which the disc inclination varies is comparable with the disc size. Also in this case, our assumptions go in the direction of underestimating the alignment timescale, since if the warp occurs over a short lengthscale even a relatively small initial misalignment might be more difficult to realign. To take this effect into account, one would need to explicitly solve for the disc shape in a time–dependent calculation (e.g. Lodato & Pringle 2006). As a consequence, we cannot quantify the exact degree of misalignment at the end of the merger, but can only perform a simple comparison between the two timescales. In order to quantify more precisely the level of misalignment, a detailed time-dependent calculation should be performed, along the lines of Perego et al. (2009) and Dotti et al. (2010). Additionally, our choice of 10 and 50 Myrs as proxies for the shrinking timescale is clearly very simplified, and might in particular be a function of the system parameters, such as the accretion rate. For example, a large value of $t_{\rm align}$ can be due to a small value of $\dot{M}$ and should then be compared with a longer shrinking timescale, suitable for the expected gas-poor environment.

Third, we have considered only the effect of gas discs on the evolution of black hole spin. To follow the evolution of the spins after decoupling due to relativistic effects, a post-Newtonian approach is needed, taking into account the possible role of spin–orbit resonances (Schnittman 2004; Berti et al. 2012).

Finally, we have considered only the alignment process of one black hole with its own disc. Aligment here is clearly a necessary condition for coalignment of the two spins in a merging binary, Even in cases where we predict alignment of individual spins and discs, the coalignment of the two spins with the larger scale circumbinary disc can present additional severe difficulties if the black holes are rapidly spinning (see Nixon & King 2012b for a discussion). All these effects act in the direction of reducing the likelihood of alignment of both spins even further.

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